

**BACCALAURÉAT GENERAL  
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES  
MATHÉMATIQUES - ANGLAIS**

**SUJET 9**

**Amicable numbers  
Theme : Arithmetics**

**Ce sujet comporte une page. L'usage de tout modèle de calculatrice, avec ou sans mode examen, est autorisé.**

There are plenty of examples of numbers with some interesting and magical properties in the history of the theory of numbers. The numbers with some unique and fascinating properties have always received a great attention in the world of mathematics. There are so many kinds of numbers like perfect numbers, amicable numbers, Fermat numbers, Fibonacci numbers, etc. which show some special characteristics. It was well said by great mathematician Kronecker (a German mathematician from the 19<sup>th</sup> century who worked on number theory, algebra and logic) that “God created the natural numbers, and all the rest is the work of man.”

Amicable numbers have the property that one number represents the other number. This symbolizes friendship, harmony, and love. The Greeks believed that these numbers had a special influence in establishing friendship between people. Amicable numbers have a huge application in astrology (in casting horoscopes) and magic.

In January 2018, there were over 1,221,159,849 known amicable pairs. But still, the search is on to discover the proper rule to find all pairs of amicable numbers. Indeed, it has not been established yet, whether the number of amicable pairs is finite or infinite. So, the search is still going on.

Adapted from the website : <https://gmsciencein.com/2018/03/12/amicable-numbers/> (28/01/19)

**I. Explain what the text deals with and comment on it.**

## II. Exercise

Two numbers are said to be amicable if the sum of the proper divisors of each number is equal to the other number (*a proper divisor of a number is any divisor of the number, except the number itself*).

For example, 1,184 and 1,210 are amicable because the proper divisors of 1,184 are 1, 2, 4, 8, 16, 32, 37, 74, 148, 296, 592 and 1,184 and the proper divisors of 1,210 are 1, 2, 5, 10, 11, 22, 55, 110, 121, 242, 605, and 1,210.

Furthermore:  $1+2+4+8+16+32+37+74+148+296+592 = 1,210$

and  $1+2+5+10+11+22+55+110+121+242+605 = 1,184$

1. a. Prove that 71 is a divisor of 284.

Therefore, find the list of all its 5 proper divisors.

b. Assuming that the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110, prove that 220 and 284 are amicable.

2. a. Find the Greatest Common Divisor (GCD) and the Lowest Common Multiple (LCM) of 1,184 and 1,210.

b. If you want to simplify the fraction  $\frac{1,184}{1,210}$ , do you need the GCD or the LCM of the two

numbers? Justify your answer and simplify the fraction.

c. If you want to add the two fractions  $\frac{65}{1,184}$  and  $\frac{153}{1,210}$ , do you need the GCD or the LCM

of 1,184 and 1,210? Explain your answer.