

**BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS**

SUJET 3

Thème : Prime numbers – Number Theory

Ce sujet comporte 1 page. L'usage de la calculatrice est autorisé.

We all know what a prime number is: it is a positive integer greater than one, in which the only divisors are one and itself. The year in which prime numbers were first studied is difficult to say. However, we know that they were of interest in the era of Pythagorus (around 500BC). Since then, many mathematicians have studied these numbers and their properties, including Euclid (around 300BC), Pierre de Fermat (in the 17th century), Leonhard Euler (in the 18th century) and even Carl Friedrich Gauss (in the 19th century). This list is not exhaustive but includes the names that any mathematician in any branch of mathematics would know.

Sophie Germain introduced a special type of prime number, now called a Sophie Germain prime. For a prime number p to be a Sophie Germain prime then both p and $2p + 1$ are prime numbers. For example, 2 is a Sophie Germain prime because both 2 and 5 are prime numbers. It is still unknown if there are infinitely many Sophie Germain primes, however, for the moment the largest known is $2\,618\,163\,402\,417 \times 2^{1\,290\,000} - 1$.

Adapted from the sites:

www-groups.dcs.st-and.ac.uk/history/HistTopics/Prime_numbers.html
mathworld.wolfram.com/SophieGermainPrime.html

1. Dégager les idées essentielles du texte ci-dessus.
2. Questions mathématiques :
 - a. In the text, six mathematicians are listed by name. Explain to the jury something that you have studied in mathematics that is related to one of these mathematicians. **Note:** it does not have to be related to prime numbers.
 - b. If n and p are both prime numbers, is the product $n \times p$ a prime number? Explain.
 - c. Is 7 a Sophie Germain prime number? What about 11?
 - d. One of many unsolved questions about prime numbers is the following: “Is there an infinity of pairs of prime numbers of which the difference is two?” For example, three and five are prime numbers and their difference is two. Five and seven would be another example. Give two more examples of this type.