

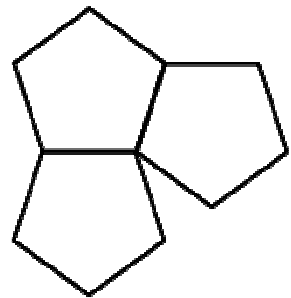
**BACCALAURÉAT GÉNÉRAL
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS**

SUJET 10

**Pentagonal tilings.
Geometry**

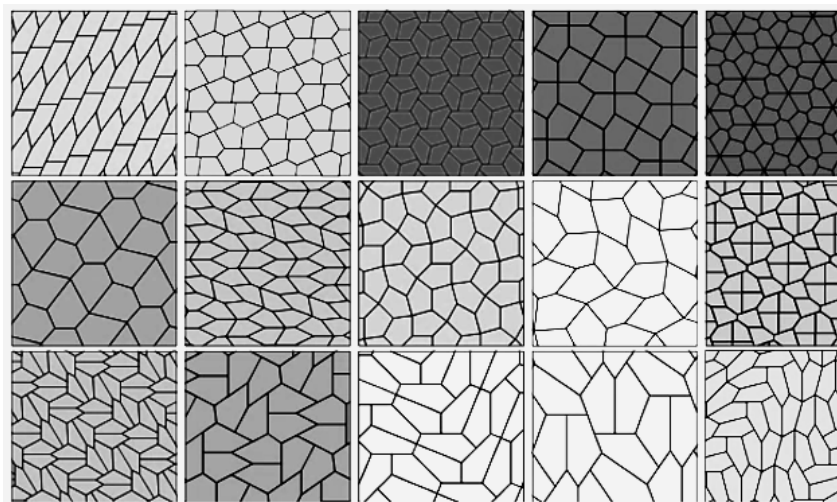
Ce sujet comporte deux pages. L'usage de tout modèle de calculatrice, avec ou sans mode examen, est autorisé, ainsi que les instruments de géométrie.

One of the oldest problems in geometry asks which shapes tile the plane. Which tiles could we choose in order to cover a flat area in an endless pattern (called a tiling), locking them together with copies of themselves like puzzle pieces? A new proof by Michaël Rao, a 37-year-old French mathematician, finally completes the classification of convex polygons that tile the plane by overcoming the last obstacle: pentagons, whose classification resisted for 99 years.



Try placing regular pentagons (those with equal angles and sides) edge to edge and gaps soon form ; they do not tile. The ancient Greeks proved that the only regular polygons that tile are triangles, quadrilaterals and hexagons. But stretch a pentagon into an irregular shape and tilings become possible.

In 1918 five types of irregular convex pentagons that tile the plane were discovered. In 1968 three more were found, and after it was published in the popular magazine Scientific American, some more were discovered by non-mathematician readers! In 2015, a 15th type of tiling with a convex pentagon was discovered. What Rao has recently proven using a computer program is that there are no more than these 15 types of pentagonal tilings.

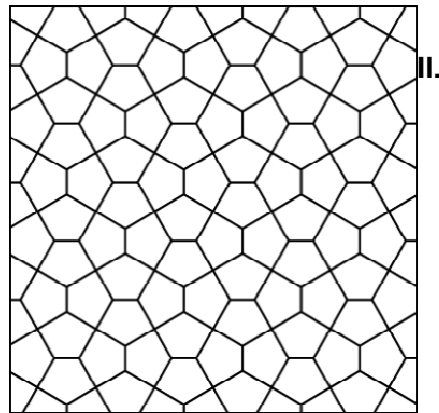


*Adapted from : Pentagon Tiling Proof Solves Century-Old Math Problem
in Quanta Magazine, by Natalie Wolchover, July 2017.*

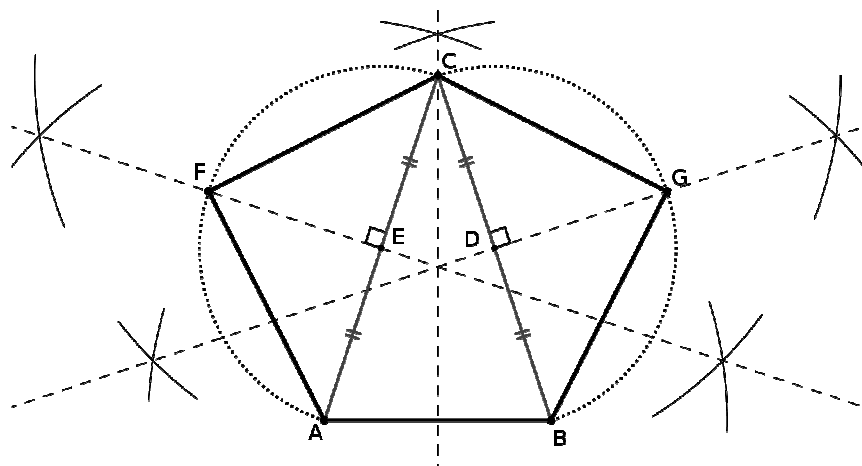
I. Explain what the text deals with and comment on it.

II. Exercise.

Let's consider this pentagonal tiling known as the Cairo tiling.



Here is a geometric construction of a possible pentagonal tile of the Cairo tiling.



1. Starting from an isosceles triangle ABC, describe step by step how to construct this pentagon ABGCF with a ruler and a pair of compasses.
2. Explain why AFC and BGC are right isosceles triangles.
3. Explain how this pentagon can be used in the Cairo tiling.
4.
 - a. How can you place the point C such that the five sides of the pentagon have the same length.
 - b. Is this pentagon regular?