Académie d'Orléans-Tours

Baccalauréat Général – Session 2011

Épreuve spécifique des sections européennes

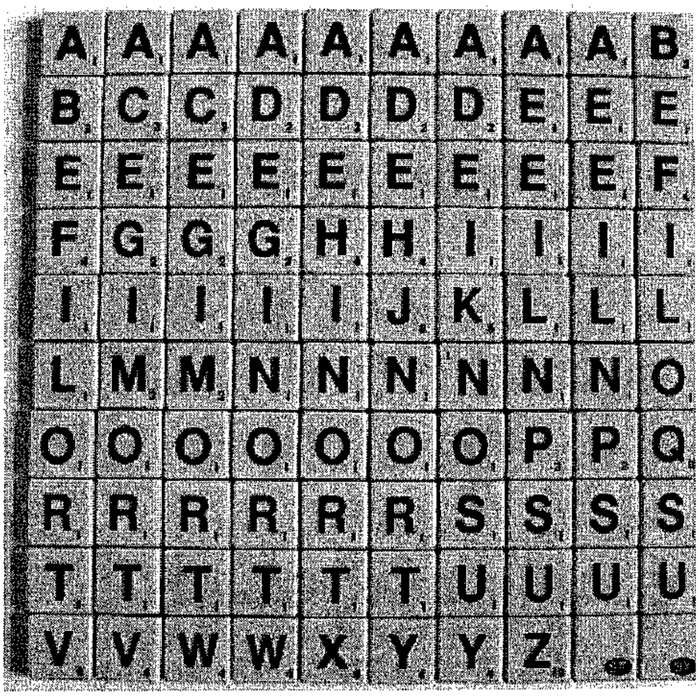
Anglais / Mathématiques

Les candidats restituent les textes à l'issue de leur épreuve.

From frequency analysis to probability: the game of Scrabble

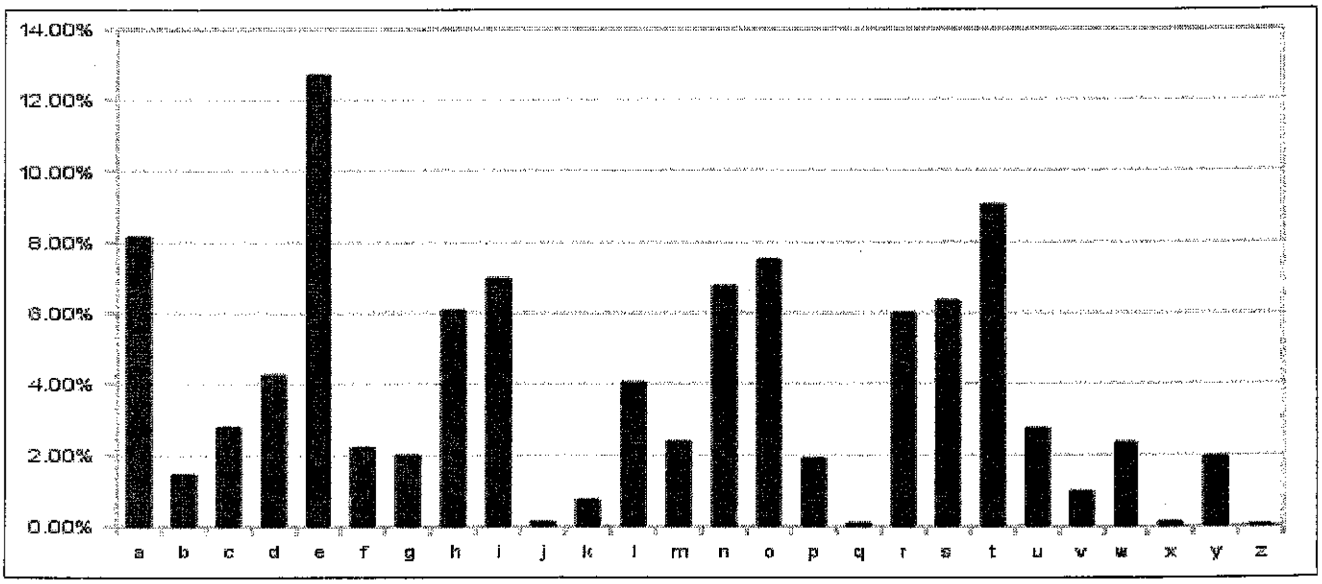
Invented by an out-of-job architect called Alfred Mosher Butts during the Great Depression, Scrabble (meaning "to scratch frantically") is a combination of vocabulary skills, anagrams, crossword puzzles and an element of chance. Butts studied the front page of *The New York Times* and did painstaking calculations of letter frequency to decide on how many tiles of each consonant and vowel to be included in the game as well as the value to be assigned to each letter. The game became hugely popular and everyone simply has to own Scrabble: one hundred and fifty million sets have been sold worldwide.

(From http://www.boardgaminginfo.com/)



The bag in a Scrabble set contains 100 tiles:

1. If you start a new game, how many chances do you have to pick a vowel?
2. The word THREE has a Scrabble score of 8. What number (less than 15) has a Scrabble score equal to its value?
3. The actual frequency of use in written English for each alphabetical letter is shown in the diagram below. Does it match the distribution of the Scrabble letters?



Newton vs Leibniz : another controversy.

Gottfried Leibniz (1646-1716) claimed that mv^2 was conserved in collisions. He called this quantity *vis viva.* His idea was that the collision transfers *vis viva* from one object to another, giving it a kind of `life' by putting it in motion.

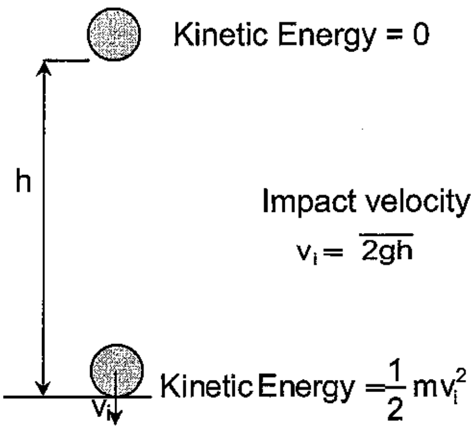
Isaac Newton (1643-1727) proved in *Principia* that mv was the conserved quantity in collisions (and conserved in general). He argued against *vis viva* conservation because it could not account for inelastic collisions.

The concept of kinetic energy, $\frac{1}{2}mv^2$, was born in a raging dispute fuelled by ego, nationalistic enthusiasm, and religious fervour. It is perhaps not surprising that the more general conservation of energy took a long time to establish. Energy can take many forms : mechanical, heat, electrical... It took centuries to explore the relationship between all these forms.

Today, the Work-Energy Principle is a very useful tool in mechanics problem solving. For a straight-line collision, this principle could be expressed as the net work done, equal to the average force of impact (F) times the distance travelled during the impact (d), being equal to the change of kinetic energy leading to the well-known formula $Fd=\frac{1}{2}\ mv_i^2\ \ mv_f^2$.

Adapted from "An introduction to the History of Science", Dr. Barbara J. Becker

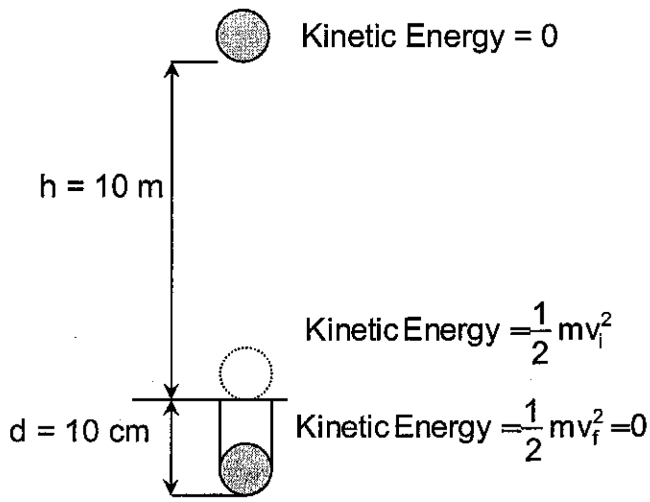
Exercise



Let's imagine the following experiment : a metallic ball of mass m is dropped from a certain height h with no initial speed. One can easily prove that the impact velocity is $v = \sqrt{2gh}$ with g being the acceleration due to gravity on Earth.
Note : In all formulae, lengths are in m, mass in Kg, velocities in $m.s^{-1}$, forces in N (newtons), $g \simeq 9.81 m.s^{-2}$.

1) Calculate the impact velocity of a 2 Kg ball dropped from a height of 1m. Do the same with a second height of 10 m and a third of 100 m.

2) Evaluate the kinetic energy of this ball just before it hits the ground for the three previous different heights. Comment your results.



3) Considering this 2 Kg ball is dropped from a 10 m height and that the hole it makes in the ground is 10 cm deep. Could you evaluate the average force of impact F of the ball on the ground?

Same question with a 5 cm deep hole. Conclude.

Same question with so small a hole you cannot really measure it. What happens to the ball in that case?

– la calculatrice est autorisée –

Which trees can you see in the forest?

On the grid below are represented the points (m,n) in the Cartesian plane where n and m are non negative integers. We call this a *lattice* of points. Imagine that each of these points represent a place where a tree is planted in a well organised forest, and that you are standing at the origin O. Then you would not be able to see every tree because some would be hidden by another tree in front of it. For instance, the tree B(4,2) is not visible from O because A(2,1) is hiding it from you. We will call A a *visible point* and B a *non visible point*.

1) Find all the visible points on the grid below and color them in.

2) Now for each point $K(m,n)$ with $m \neq 0$, consider the fraction n/m.

   a) What does n/m represent for the line (OK) ?

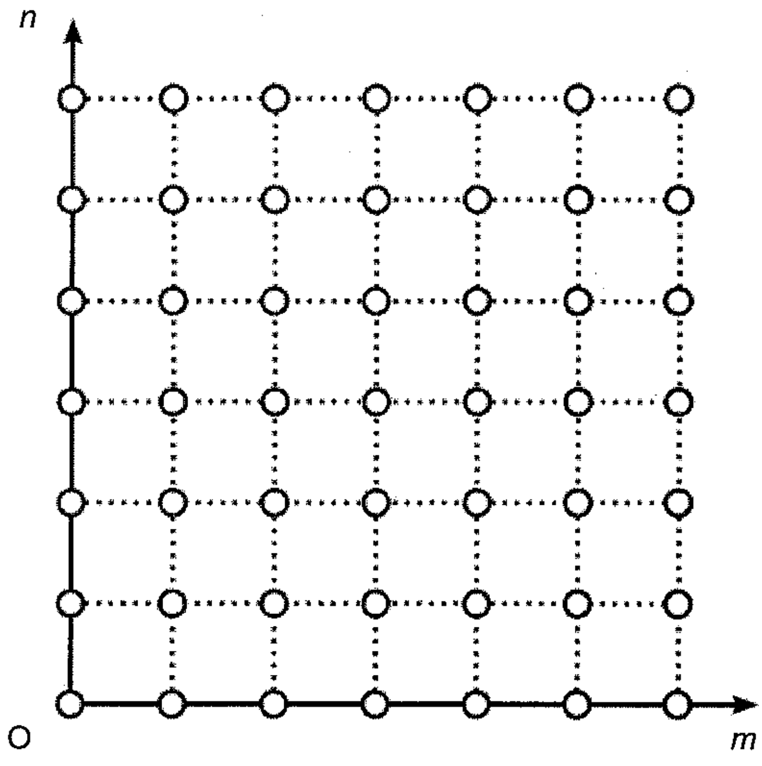
   b) Explain why the visible points are those for which the fraction n/m is irreducible.

3) Write down those 11 fractions with $n \leq m \leq 5$ in increasing order:

   0/1, 1/5, 1/4, 1/3, 2/5,..,1/1.

This is the Farey serie of order 5.

Find the Farey serie of order 6.



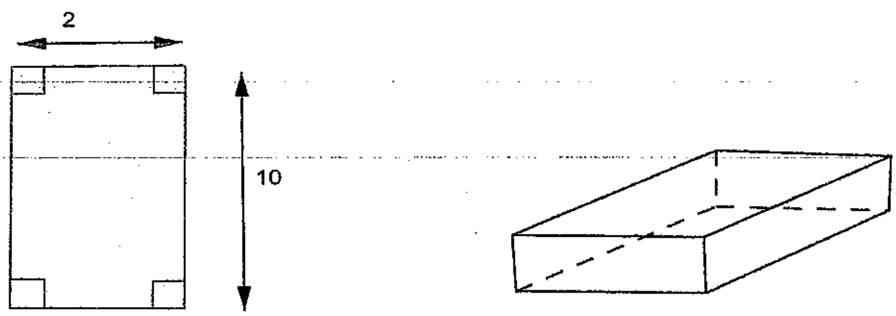
<u>Functions</u>

As a mathematical term, "**function**" was coined by Gottfried Leibniz, in a 1673 letter, to describe a quantity related to a curve, such as a curve's slope at a specific point. The functions Leibniz considered are today called differentiable functions. For this type of function, one can talk about limits and derivatives; both are measurements of the output or the change in the output as it depends on the input or the change in the input. Such functions are the basis of calculus.
Analytical functions were also introduced by Isaac Newton in a notation which he used to solve problems of mathematical physics. He used the methods of calculus to solve the problem of planetary motion, the shape of the surface of a rotating fluid, and many other problems discussed in his *Principia Mathematica* (1687).
There are many ways to describe or represent functions: by a formula, by an algorithm that computes it, by a plot or a graph. A table of values is a common way to specify a function in statistics, physics, chemistry, and other sciences.

http://en.wikipedia.org/wiki/Calculus (and Functions)

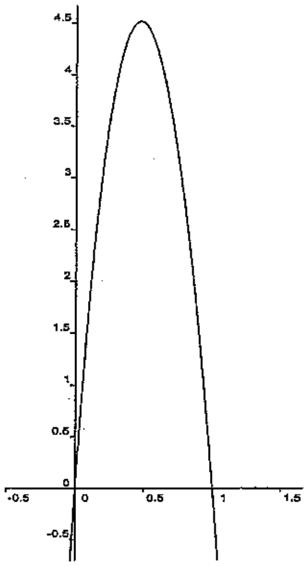
<u>Exercise :</u>

A company wants to make an open box to put scent bottles in. A sheet of cardboard 10 inches by 2 inches is used to make it. Squares of equal sides x are cut out of each corner. Then, the sides are folded to make the box.



The company wants to know the value of x that makes the maximum volume .

1) Write the volume of the box. We'll note it V(x).
2) What are the possible values of x ?
3) Choose two values for x, and work out the volume of the box for them.



4) This is the graph of the function V.
   Use the graph to :
   a) Re-check question 3.
   b) Find the value of x that makes the maximum volume.

Archimedes' recipe for pi:

Text:

A little known verse of the Bible, also found in a list of specifications for the great temple of Solomon built around 950 BC, states that a good approximation of π is 3. This is not a very accurate value of course and not even very accurate in its day, for the Egyptian and *Mesopotamian* values of $^{25}/_8$ = 3.125 and √10 = 3.162 have been traced to much earlier dates.

The fact that the ratio of the circumference to the diameter of a circle is constant has been known for so long that it is quite untraceable. The earliest values of π including the 'Biblical' value of 3, were almost certainly found by measurement. In the Egyptian Rhind Papyrus, which is dated about 1650 BC, there is good evidence for 3.16 as a value for π.

Exercise:

The first theoretical calculation seems to have been carried out by Archimedes of Syracuse (287-212 BC). Consider a circle of radius 1, in which we inscribe a regular polygon of 3 x 2^{n-1} sides, with semiperimeter b_n, and superscribe a regular polygon of 3 x 2^{n-1} sides, with semiperimeter a_n.

Archimedes obtained the approximation : $\frac{223}{71} < \pi < \frac{22}{7}$ (*).

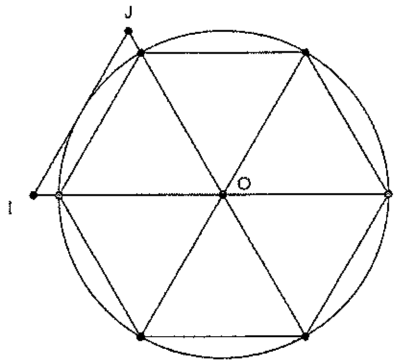
http://www-history.mcs.st-and.ac.uk/HistTopics/Pi_through_the_ages.html

1) We take n=2. The number of sides of the polygon is 6.

a) Execute the following instructions to trace the superscribed hexagon in the figure 1

'*Connect the lines around the outside of the circle to create a second hexagon that just touches the circle's outside edge. Make sure that the straight line for each segment touches the circle at the segment's halfway point.*'

Figure 1:



b) In the figure 2, the triangle OJI is isosceles, G is the midpoint of [JI] and OG=1. Calculate IG using tan $(\widehat{OGI})$

c) Deduce from the calculation of IG the perimeter of the superscribed hexagon. It is the upper bound for the approximation of Pi with n=2.

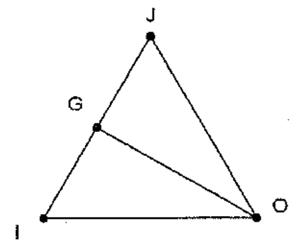


Figure 2

2) We take Archimedes' estimate of Pi for a given value of n as the average of the two bounds. How much is the error made if we compare it to the usual approximation Pi≈3.14159? Complete the table and make some comments about this error.

Value of n	2	3	4	5	6
Number of sides	6	12	24	48	96
Lower bound a_n	3	3.106	3.133	3.1393	223/71=3.1408...
Upper bound b_n	3.46	3.249	3.159	3.1461	22/7=3.1429...

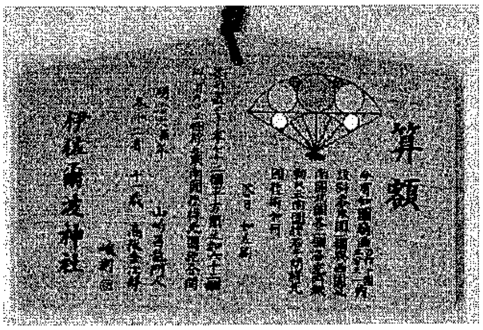
L'usage de la calculatrice est autorisé

Have you ever heard about *sangaku* ?

I. Introduction

Of the world's countless customs and traditions, perhaps none is as elegant, nor as beautiful, as the tradition of *sangaku*, Japanese temple geometry.

From 1639 to 1854, Japan lived in strict, self-imposed isolation from the West. Access to all forms of <u>Western</u> culture was suppressed. During this period, a kind of native mathematics flourished.



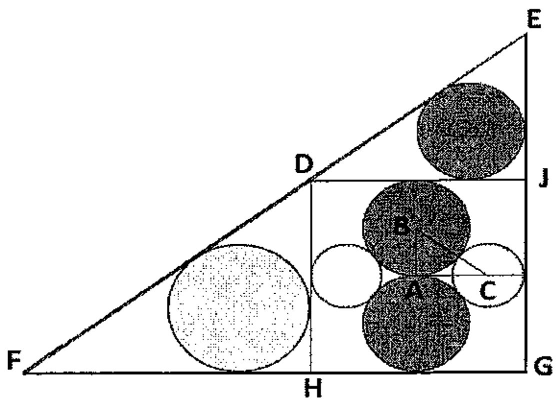
Devotees of math, evidently samurai, merchants and farmers, would solve a wide variety of geometry problems, inscribe their efforts in delicately colored wooden tablets and hang the works under the roofs of religious buildings. These sangaku, a word that literally means *mathematical tablet*, may have been acts of homage--a thank to a guiding spirit--or they may have been brazen challenges to other worshipers: Solve this one if you can! For the most part, sangaku deal with ordinary Euclidean geometry. But the problems are strikingly different from those found in a typical high school geometry course. Circles and ellipses play a far more prominent role than in Western problems: circles within ellipses, ellipses within circles. Some of the exercises are quite simple and could be solved by first-year students. Others are nearly impossible, and modern geometers invariably tackle them with advanced methods, including calculus and affine transformations.

II. Questions

I. Could you explain what the word Sangaku means, and where it comes from ?

II. What were those mathematics dealing with in most cases ?

Two dark grey circles of radius r and white circles of radius t are inscribed in a square, as shown. The square itself is inscribed in a large triangle and, as illustrated, two circles of radii R and r are inscribed in the small triangles outside the square.
The goal of this exercise is to show that $R=2t$.



III. Find out the side of the square.

IV. Applying one famous geometry theorem in the right triangle ABC, shows that $r=\frac{3t}{2}$.

V. Assuming that $DE=5r$, show the short side of the upper right triangle equals to 3*r*.

VI. Explain why the three right triangles EFG, FDH and DEJ have the proportions 3-4-5.

VII. Shows that $4r=3R$ *and* finally, $R=\frac{4r}{3}=2t$.

Source : http://www.cut-the-knot.org/pythagoras/Sangaku.shtml

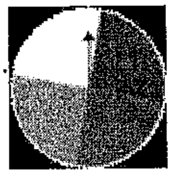
Probabilités

PART N°1

Definitions:
An **experiment** is a situation involving chance or probability that leads to results called outcomes.
An **outcome** is the result of a single trial of an experiment.
An **event** is one or more outcomes of an experiment.
Probability is the measure of how likely an event is.
Equiprobability is a concept that allows one to assign equal probabilities to outcomes when they are judged to be equipossible or to be "equally likely" in some sense.

In order to measure probabilities, mathematicians have devised the following formula for finding the probability of an event: it is given by the number of times that event can occur divided by the total number of outcomes.

Problem: A spinner has 4 equal sectors colored yellow, blue, green and red.



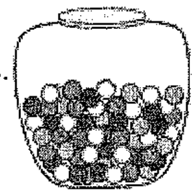
Questions:
In the problem above:
1) Describe the experiment.
2) What are all the possible outcomes?
3) Describe one event that is not an outcome.
4) What is the probability of landing on blue after spinning the spinner?
5) Do you think this experiment can be called equiprobable?

PART N°2

Experiment A: A single 6-sided die is rolled.

1) Describe the different outcomes.
2) What is the probability of each outcome?
3) Which of the definitions above would you choose to describe « rolling an even number »?What is the probability of rolling an even number?

Experiment B: A glass jar contains 6 red, 7 green, 8 blue and 3 yellow marbles. A single marble is chosen at random from the jar.



1) What is the probability of choosing a green marble?
2) a blue marble?
3) a yellow marble?
4) a red marble? Give all the different ways of answering.
5) Is is a situation of equiprobability?

Bode's Law

The average distances of the planets from the sun display a pattern first described by Johann Bode in 1772. This relationship is called Bode's Law and was proposed before Uranus, Neptune and Pluto were discovered. It is a sequence defined by :

$$a_1 = 0.4$$
$$a_n = 0.3 \times 2^{n-2} + 0.4$$
$$\text{for } n = 2, 3, 4, \dots, 9.$$

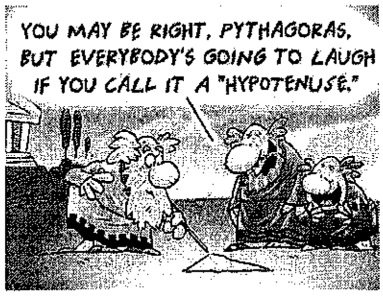
In this sequence, a distance of 1 unit corresponds to the average Earth-Sun distance of 93 million miles. The number n represents the n^{th} planet. The actual distances of the planets, including an average distance for the asteroïds, are listed in the accompanying table. *(Source: M. Zeilik, introductory Astronomy and Astrophysics.)*

a) Is Bode's sequence arithmetic, geometric or neither of those ? Why ?
b) Find a_4 and interpret the result.
c) Calculate the terms of Bode's sequence. Compare them with the values in the table.
d) If there is another planet beyond Neptune, use Bode's law to predict its distance from the sun.

Planet	Distance
Mercury	0.39
Venus	0.72
Earth	1.00
Mars	1.52
Asteroïds	2.8
Jupiter	5.20
Saturn	9.54
Uranus	19.2
Neptune	30.1

English oral test

I Text



The Ancient Greeks assumed that any length or area they calculated would be rational -that is that it could be expressed by a fraction. The Pythagoreans were fascinated by right-angled triangles : they measured the diagonal of a square of unit side and discovered that they could not express this length as a rational number. Ancient Greek mathematician Hippasus of Metapontum, who was a dedicated Pythagorean, used the Pythagorean theorem to prove that the diagonal of a square with rational sides does not have a rational length.

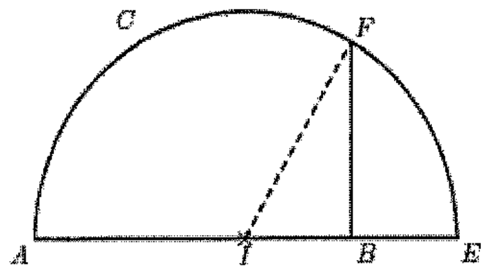
The hypotenuse of a right-angled triangle with sides of one unit is around 1.4141... This is an irrational number known as « Pythagoras' Constant». It cannot be written as a fraction and is a recurring decimal number with no pattern of repetition.

From : « More BrainMatics Logic Puzzles » by Ivan Moscovitch

II Exercise

The aim of the graph below is to show an easy way to draw the square root of a positive number *a*, using only a ruler and a compass.

- [AB] is a segment-line of length *a*.
- The length of [BE] is 1.
- The circle *C* is centered at the middle I of [AE].
- EBF is a right triangle in B.



1. Work out the length AI and then deduce the value of the length IB.
2. What is the distance between the points I and F ? Explain why.
3. What famous theorem can you use in the triangle IBF and why ?
4. Show that the length of the segment line [BF] is $\sqrt{a}$.
5. A segment line of length 1 is given below.

Explain how to draw a segment-line of length $\sqrt{5,5}$ with a ruler and a compass.